Integration by Parts

✔ QUICK CHECK EXERCISES 7.2 (See page 500 for answers.)

1. (a) If $G'(x) = g(x)$, then
$$\int f(x)g(x)\,dx = f(x)G(x) - \_\_\_\_\_\_$$
(b) If $u = f(x)$ and $v = G(x)$, then the formula in part (a) can be written in the form $\int u\,dv = \_\_\_\_\_\_$.

2. Find an appropriate choice of u and dv for integration by parts of each integral. Do not evaluate the integral.
(a) $\int x \ln x\,dx$; $u = \_\_\_\_\_\_$, $dv = \_\_\_\_\_\_$
(b) $\int (x-2)\sin x\,dx$; $u = \_\_\_\_\_\_$, $dv = \_\_\_\_\_\_$
(c) $\int \sin^{-1} x\,dx$; $u = \_\_\_\_\_\_$, $dv = \_\_\_\_\_\_$
(d) $\int \frac{x}{\sqrt{x-1}}\,dx$; $u = \_\_\_\_\_\_$, $dv = \_\_\_\_\_\_$

3. Use integration by parts to evaluate the integral.
(a) $\int xe^{2x}\,dx$ (b) $\int \ln(x-1)\,dx$
(c) $\int_0^{\pi/6} x \sin 3x\,dx$

4. Use a reduction formula to evaluate $\int \sin^3 x\,dx$.

EXERCISE SET 7.2

1–38 Evaluate the integral.

1. $\int xe^{-2x}\,dx$
2. $\int xe^{3x}\,dx$
3. $\int x^2e^{x}\,dx$
4. $\int x^2e^{-2x}\,dx$
5. $\int x\sin 3x\,dx$
6. $\int x\cos 2x\,dx$
7. $\int x^2\cos x\,dx$
8. $\int x^2\sin x\,dx$
9. $\int x\ln x\,dx$
10. $\int \sqrt{x}\ln x\,dx$
11. $\int (\ln x)^2\,dx$
12. $\int \frac{\ln x}{\sqrt{x}}\,dx$
13. $\int \ln(3x-2)\,dx$
14. $\int \ln(x^2+4)\,dx$
15. $\int \sin^{-1} x\,dx$
16. $\int \cos^{-1}(2x)\,dx$
17. $\int \tan^{-1}(3x)\,dx$
18. $\int x\tan^{-1} x\,dx$
19. $\int e^{x}\sin x\,dx$
20. $\int e^{3x}\cos 2x\,dx$
21. $\int \sin(\ln x)\,dx$
22. $\int \cos(\ln x)\,dx$
23. $\int x\sec^2 x\,dx$
24. $\int x\tan^2 x\,dx$
25. $\int x^3e^{x^2}\,dx$
26. $\int \frac{xe^{x}}{(x+1)^2}\,dx$
27. $\int_0^2 xe^{2x}\,dx$
28. $\int_0^1 xe^{-5x}\,dx$
29. $\int_1^e x^2\ln x\,dx$
30. $\int_{\sqrt{e}}^{e} \frac{\ln x}{x^2}\,dx$
31. $\int_{-1}^{1} \ln(x+2)\,dx$
32. $\int_0^{\sqrt{3}/2} \sin^{-1} x\,dx$
33. $\int_2^4 \sec^{-1}\sqrt{\theta}\,d\theta$
34. $\int_1^2 x\sec^{-1} x\,dx$
35. $\int_0^{\pi} x\sin 2x\,dx$
36. $\int_0^{\pi} (x + x\cos x)\,dx$
37. $\int_1^3 \sqrt{x}\tan^{-1}\sqrt{x}\,dx$
38. $\int_0^2 \ln(x^2+1)\,dx$

39–42 True–False Determine whether the statement is true or false. Explain your answer.

39. The main goal in integration by parts is to choose u and dv to obtain a new integral that is easier to evaluate than the original.

40. Applying the LIATE strategy to evaluate $\int x^3 \ln x\,dx$, we should choose $u = x^3$ and $dv = \ln x\,dx$.

41. To evaluate $\int \ln e^x\,dx$ using integration by parts, choose $dv = e^x\,dx$.

42. Tabular integration by parts is useful for integrals of the form $\int p(x)f(x)\,dx$, where $p(x)$ is a polynomial and $f(x)$ can be repeatedly integrated.

43–44 Evaluate the integral by making a u-substitution and then integrating by parts.

43. $\int e^{\sqrt{x}}\,dx$
44. $\int \cos\sqrt{x}\,dx$

45. Prove that tabular integration by parts gives the correct answer for
$$\int p(x)f(x)\,dx$$
where $p(x)$ is any *quadratic polynomial* and $f(x)$ is any function that can be repeatedly integrated.

46. The computations of any integral evaluated by repeated integration by parts can be organized using tabular integration by parts. Use this organization to evaluate $\int e^x \cos x\,dx$ in

two ways: first by repeated differentiation of $\cos x$ (compare Example 5), and then by repeated differentiation of e^x.

47–52 Evaluate the integral using tabular integration by parts.

47. $\int (3x^2 - x + 2)e^{-x}\,dx$ **48.** $\int (x^2 + x + 1)\sin x\,dx$

49. $\int 4x^4 \sin 2x\,dx$ **50.** $\int x^3\sqrt{2x+1}\,dx$

51. $\int e^{ax}\sin bx\,dx$ **52.** $\int e^{-3\theta}\sin 5\theta\,d\theta$

53. Consider the integral $\int \sin x \cos x\,dx$.
(a) Evaluate the integral two ways: first using integration by parts, and then using the substitution $u = \sin x$.
(b) Show that the results of part (a) are equivalent.
(c) Which of the two methods do you prefer? Discuss the reasons for your preference.

54. Evaluate the integral

$$\int_0^1 \frac{x^3}{\sqrt{x^2+1}}\,dx$$

using
(a) integration by parts
(b) the substitution $u = \sqrt{x^2+1}$.

55. (a) Find the area of the region enclosed by $y = \ln x$, the line $x = e$, and the x-axis.
(b) Find the volume of the solid generated when the region in part (a) is revolved about the x-axis.

56. Find the area of the region between $y = x\sin x$ and $y = x$ for $0 \le x \le \pi/2$.

57. Find the volume of the solid generated when the region between $y = \sin x$ and $y = 0$ for $0 \le x \le \pi$ is revolved about the y-axis.

58. Find the volume of the solid generated when the region enclosed between $y = \cos x$ and $y = 0$ for $0 \le x \le \pi/2$ is revolved about the y-axis.

59. A particle moving along the x-axis has velocity function $v(t) = t^3 \sin t$. How far does the particle travel from time $t = 0$ to $t = \pi$?

60. The study of sawtooth waves in electrical engineering leads to integrals of the form

$$\int_{-\pi/\omega}^{\pi/\omega} t\sin(k\omega t)\,dt$$

where k is an integer and ω is a nonzero constant. Evaluate the integral.

61. Use reduction formula (9) to evaluate
(a) $\int \sin^4 x\,dx$ (b) $\int_0^{\pi/2} \sin^5 x\,dx$.

62. Use reduction formula (10) to evaluate
(a) $\int \cos^5 x\,dx$ (b) $\int_0^{\pi/2} \cos^6 x\,dx$.

63. Derive reduction formula (9).

64. In each part, use integration by parts or other methods to derive the reduction formula.

(a) $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2}x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx$

(b) $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx$

(c) $\displaystyle\int x^n e^x\,dx = x^n e^x - n\int x^{n-1}e^x\,dx$

65–66 Use the reduction formulas in Exercise 64 to evaluate the integrals.

65. (a) $\int \tan^4 x\,dx$ (b) $\int \sec^4 x\,dx$ (c) $\int x^3 e^x\,dx$

66. (a) $\int x^2 e^{3x}\,dx$ (b) $\int_0^1 xe^{-\sqrt{x}}\,dx$
[*Hint:* First make a substitution.]

67. Let f be a function whose second derivative is continuous on $[-1, 1]$. Show that

$$\int_{-1}^{1} xf''(x)\,dx = f'(1) + f'(-1) - f(1) + f(-1)$$

FOCUS ON CONCEPTS

68. (a) In the integral $\int x\cos x\,dx$, let

$$u = x, \quad dv = \cos x\,dx,$$
$$du = dx, \quad v = \sin x + C_1$$

Show that the constant C_1 cancels out, thus giving the same solution obtained by omitting C_1.
(b) Show that in general

$$uv - \int v\,du = u(v + C_1) - \int (v + C_1)\,du$$

thereby justifying the omission of the constant of integration when calculating v in integration by parts.

69. Evaluate $\int \ln(x+1)\,dx$ using integration by parts. Simplify the computation of $\int v\,du$ by introducing a constant of integration $C_1 = 1$ when going from dv to v.

70. Evaluate $\int \ln(3x-2)\,dx$ using integration by parts. Simplify the computation of $\int v\,du$ by introducing a constant of integration $C_1 = -\frac{2}{3}$ when going from dv to v. Compare your solution with your answer to Exercise 13.

71. Evaluate $\int x\tan^{-1}x\,dx$ using integration by parts. Simplify the computation of $\int v\,du$ by introducing a constant of integration $C_1 = \frac{1}{2}$ when going from dv to v.

72. What equation results if integration by parts is applied to the integral

$$\int \frac{1}{x\ln x}\,dx$$

with the choices

$$u = \frac{1}{\ln x} \quad \text{and} \quad dv = \frac{1}{x}\,dx?$$

In what sense is this equation true? In what sense is it false?

✔ QUICK CHECK ANSWERS 7.2

1. (a) $\int f'(x)G(x)\,dx$ (b) $uv - \int v\,du$ 2. (a) $\ln x$; $x\,dx$ (b) $x-2$; $\sin x\,dx$ (c) $\sin^{-1} x$; dx (d) x; $\dfrac{1}{\sqrt{x-1}}\,dx$
3. (a) $\left(\dfrac{x}{2}-\dfrac{1}{4}\right)e^{2x}+C$ (b) $(x-1)\ln(x-1)-x+C$ (c) $\frac{1}{9}$ 4. $-\frac{1}{3}\sin^2 x\cos x - \frac{2}{3}\cos x + C$

Trigonometric Integrals

✔ QUICK CHECK EXERCISES 7.3

(See page 508 for answers.)

1. Complete each trigonometric identity with an expression involving $\cos 2x$.
(a) $\sin^2 x =$ ________ (b) $\cos^2 x =$ ________
(c) $\cos^2 x - \sin^2 x =$ ________

2. Evaluate the integral.
(a) $\int \sec^2 x\,dx =$ ________
(b) $\int \tan^2 x\,dx =$ ________
(c) $\int \sec x\,dx =$ ________
(d) $\int \tan x\,dx =$ ________

3. Use the indicated substitution to rewrite the integral in terms of u. Do not evaluate the integral.
(a) $\int \sin^2 x\cos x\,dx$; $u = \sin x$
(b) $\int \sin^3 x\cos^2 x\,dx$; $u = \cos x$
(c) $\int \tan^3 x\sec^2 x\,dx$; $u = \tan x$
(d) $\int \tan^3 x\sec x\,dx$; $u = \sec x$

EXERCISE SET 7.3

1–52 Evaluate the integral. ■

1. $\int \cos^3 x\sin x\,dx$ **2.** $\int \sin^5 3x\cos 3x\,dx$

3. $\int \sin^2 5\theta\,d\theta$ **4.** $\int \cos^2 3x\,dx$

5. $\int \sin^3 a\theta\,d\theta$ **6.** $\int \cos^3 at\,dt$

7. $\int \sin ax\cos ax\,dx$ **8.** $\int \sin^3 x\cos^3 x\,dx$

9. $\int \sin^2 t\cos^3 t\,dt$ **10.** $\int \sin^3 x\cos^2 x\,dx$

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11. $\int \sin^2 x \cos^2 x \, dx$

12. $\int \sin^2 x \cos^4 x \, dx$

13. $\int \sin 2x \cos 3x \, dx$

14. $\int \sin 3\theta \cos 2\theta \, d\theta$

15. $\int \sin x \cos(x/2) \, dx$

16. $\int \cos^{1/3} x \sin x \, dx$

17. $\int_0^{\pi/2} \cos^3 x \, dx$

18. $\int_0^{\pi/2} \sin^2 \frac{x}{2} \cos^2 \frac{x}{2} \, dx$

19. $\int_0^{\pi/3} \sin^4 3x \cos^3 3x \, dx$

20. $\int_{-\pi}^{\pi} \cos^2 5\theta \, d\theta$

21. $\int_0^{\pi/6} \sin 4x \cos 2x \, dx$

22. $\int_0^{2\pi} \sin^2 kx \, dx$

23. $\int \sec^2(2x-1) \, dx$

24. $\int \tan 5x \, dx$

25. $\int e^{-x} \tan(e^{-x}) \, dx$

26. $\int \cot 3x \, dx$

27. $\int \sec 4x \, dx$

28. $\int \frac{\sec(\sqrt{x})}{\sqrt{x}} \, dx$

29. $\int \tan^2 x \sec^2 x \, dx$

30. $\int \tan^5 x \sec^4 x \, dx$

31. $\int \tan 4x \sec^4 4x \, dx$

32. $\int \tan^4 \theta \sec^4 \theta \, d\theta$

33. $\int \sec^5 x \tan^3 x \, dx$

34. $\int \tan^5 \theta \sec \theta \, d\theta$

35. $\int \tan^4 x \sec x \, dx$

36. $\int \tan^2 x \sec^3 x \, dx$

37. $\int \tan t \sec^3 t \, dt$

38. $\int \tan x \sec^5 x \, dx$

39. $\int \sec^4 x \, dx$

40. $\int \sec^5 x \, dx$

41. $\int \tan^3 4x \, dx$

42. $\int \tan^4 x \, dx$

43. $\int \sqrt{\tan x} \sec^4 x \, dx$

44. $\int \tan x \sec^{3/2} x \, dx$

45. $\int_0^{\pi/8} \tan^2 2x \, dx$

46. $\int_0^{\pi/6} \sec^3 2\theta \tan 2\theta \, d\theta$

47. $\int_0^{\pi/2} \tan^5 \frac{x}{2} \, dx$

48. $\int_0^{1/4} \sec \pi x \tan \pi x \, dx$

49. $\int \cot^3 x \csc^3 x \, dx$

50. $\int \cot^2 3t \sec 3t \, dt$

51. $\int \cot^3 x \, dx$

52. $\int \csc^4 x \, dx$

53–56 True–False Determine whether the statement is true or false. Explain your answer.

53. To evaluate $\int \sin^5 x \cos^8 x \, dx$, use the trigonometric identity $\sin^2 x = 1 - \cos^2 x$ and the substitution $u = \cos x$.

54. To evaluate $\int \sin^8 x \cos^5 x \, dx$, use the trigonometric identity $\sin^2 x = 1 - \cos^2 x$ and the substitution $u = \cos x$.

55. The trigonometric identity

$$\sin \alpha \cos \beta = \tfrac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

is often useful for evaluating integrals of the form $\int \sin^m x \cos^n x \, dx$.

56. The integral $\int \tan^4 x \sec^5 x \, dx$ is equivalent to one whose integrand is a polynomial in $\sec x$.

57. Let m, n be distinct nonnegative integers. Use Formulas (16)–(18) to prove:

(a) $\int_0^{2\pi} \sin mx \cos nx \, dx = 0$

(b) $\int_0^{2\pi} \cos mx \cos nx \, dx = 0$

(c) $\int_0^{2\pi} \sin mx \sin nx \, dx = 0$.

58. Evaluate the integrals in Exercise 57 when m and n denote the *same* nonnegative integer.

59. Find the arc length of the curve $y = \ln(\cos x)$ over the interval $[0, \pi/4]$.

60. Find the volume of the solid generated when the region enclosed by $y = \tan x$, $y = 1$, and $x = 0$ is revolved about the x-axis.

61. Find the volume of the solid that results when the region enclosed by $y = \cos x$, $y = \sin x$, $x = 0$, and $x = \pi/4$ is revolved about the x-axis.

62. The region bounded below by the x-axis and above by the portion of $y = \sin x$ from $x = 0$ to $x = \pi$ is revolved about the x-axis. Find the volume of the resulting solid.

63. Use Formula (27) to show that if the length of the equatorial line on a Mercator projection is L, then the vertical distance D between the latitude lines at $\alpha°$ and $\beta°$ on the same side of the equator (where $\alpha < \beta$) is

$$D = \frac{L}{2\pi} \ln \left| \frac{\sec \beta° + \tan \beta°}{\sec \alpha° + \tan \alpha°} \right|$$

64. Suppose that the equator has a length of 100 cm on a Mercator projection. In each part, use the result in Exercise 63 to answer the question.

(a) What is the vertical distance on the map between the equator and the line at 25° north latitude?

(b) What is the vertical distance on the map between New Orleans, Louisiana, at 30° north latitude and Winnipeg, Canada, at 50° north latitude?

FOCUS ON CONCEPTS

65. (a) Show that

$$\int \csc x \, dx = -\ln|\csc x + \cot x| + C$$

(b) Show that the result in part (a) can also be written as

$$\int \csc x \, dx = \ln|\csc x - \cot x| + C$$

and

$$\int \csc x \, dx = \ln\left|\tan \tfrac{1}{2}x\right| + C$$

66. Rewrite $\sin x + \cos x$ in the form

$$A \sin(x + \phi)$$

and use your result together with Exercise 65 to evaluate

$$\int \frac{dx}{\sin x + \cos x}$$

67. Use the method of Exercise 66 to evaluate

$$\int \frac{dx}{a \sin x + b \cos x} \quad (a, b \text{ not both zero})$$

68. (a) Use Formula (9) in Section 7.2 to show that

$$\int_0^{\pi/2} \sin^n x\, dx = \frac{n-1}{n} \int_0^{\pi/2} \sin^{n-2} x\, dx \quad (n \geq 2)$$

(b) Use this result to derive the ***Wallis sine formulas***:

$$\int_0^{\pi/2} \sin^n x\, dx = \frac{\pi}{2} \cdot \frac{1 \cdot 3 \cdot 5 \cdots (n-1)}{2 \cdot 4 \cdot 6 \cdots n} \quad \begin{pmatrix} n \text{ even} \\ \text{and} \geq 2 \end{pmatrix}$$

$$\int_0^{\pi/2} \sin^n x\, dx = \frac{2 \cdot 4 \cdot 6 \cdots (n-1)}{3 \cdot 5 \cdot 7 \cdots n} \quad \begin{pmatrix} n \text{ odd} \\ \text{and} \geq 3 \end{pmatrix}$$

69. Use the Wallis formulas in Exercise 68 to evaluate

(a) $\displaystyle\int_0^{\pi/2} \sin^3 x\, dx$ (b) $\displaystyle\int_0^{\pi/2} \sin^4 x\, dx$

(c) $\displaystyle\int_0^{\pi/2} \sin^5 x\, dx$ (d) $\displaystyle\int_0^{\pi/2} \sin^6 x\, dx.$

70. Use Formula (10) in Section 7.2 and the method of Exercise 68 to derive the ***Wallis cosine formulas***:

$$\int_0^{\pi/2} \cos^n x\, dx = \frac{\pi}{2} \cdot \frac{1 \cdot 3 \cdot 5 \cdots (n-1)}{2 \cdot 4 \cdot 6 \cdots n} \quad \begin{pmatrix} n \text{ even} \\ \text{and} \geq 2 \end{pmatrix}$$

$$\int_0^{\pi/2} \cos^n x\, dx = \frac{2 \cdot 4 \cdot 6 \cdots (n-1)}{3 \cdot 5 \cdot 7 \cdots n} \quad \begin{pmatrix} n \text{ odd} \\ \text{and} \geq 3 \end{pmatrix}$$

71. **Writing** Describe the various approaches for evaluating integrals of the form

$$\int \sin^m x \cos^n x\, dx$$

Into what cases do these types of integrals fall? What procedures and identities are used in each case?

72. **Writing** Describe the various approaches for evaluating integrals of the form

$$\int \tan^m x \sec^n x\, dx$$

Into what cases do these types of integrals fall? What procedures and identities are used in each case?

✔ QUICK CHECK ANSWERS 7.3

1. (a) $\dfrac{1 - \cos 2x}{2}$ (b) $\dfrac{1 + \cos 2x}{2}$ (c) $\cos 2x$ **2.** (a) $\tan x + C$ (b) $\tan x - x + C$ (c) $\ln|\sec x + \tan x| + C$ (d) $\ln|\sec x| + C$
3. (a) $\displaystyle\int u^2\, du$ (b) $\displaystyle\int (u^2 - 1)u^2\, du$ (c) $\displaystyle\int u^3\, du$ (d) $\displaystyle\int (u^2 - 1)\, du$

Answer

Exercise Set 7.2 (Page 498)

1. $-e^{-2x}\left(\frac{x}{2}+\frac{1}{4}\right)+C$ **3.** $x^2e^x-2xe^x+2e^x+C$

5. $-\frac{1}{3}x\cos 3x+\frac{1}{9}\sin 3x+C$ **7.** $x^2\sin x+2x\cos x-2\sin x+C$

9. $\frac{x^2}{2}\ln x-\frac{x^2}{4}+C$ **11.** $x(\ln x)^2-2x\ln x+2x+C$

13. $x\ln(3x-2)-x-\frac{2}{3}\ln(3x-2)+C$ **15.** $x\sin^{-1}x+\sqrt{1-x^2}+C$

17. $x\tan^{-1}(3x)-\frac{1}{6}\ln(1+9x^2)+C$ **19.** $\frac{1}{2}e^x(\sin x-\cos x)+C$

21. $(x/2)[\sin(\ln x)-\cos(\ln x)]+C$ **23.** $x\tan x+\ln|\cos x|+C$

25. $\frac{1}{2}x^2e^{x^2}-\frac{1}{2}e^{x^2}+C$ **27.** $\frac{1}{4}(3e^4+1)$ **29.** $(2e^3+1)/9$

31. $3\ln 3-2$ **33.** $\frac{5\pi}{6}-\sqrt{3}+1$ **35.** $-\pi/2$

37. $\frac{1}{3}\left(2\sqrt{3}\pi-\frac{\pi}{2}-2+\ln 2\right)$

Responses to True–False questions may be abridged to save space.

39. True; see the subsection "Guidelines for Integration by Parts."

41. False; e^x isn't a factor of the integrand.

43. $2(\sqrt{x}-1)e^{\sqrt{x}}+C$ **47.** $-(3x^2+5x+7)e^{-x}+C$

49. $(4x^3-6x)\sin 2x-(2x^4-6x^2+3)\cos 2x+C$

51. $\frac{e^{ax}}{a^2+b^2}(a\sin bx-b\cos bx)+C$ **53.** (a) $\frac{1}{2}\sin^2 x+C$

55. (a) $A=1$ (b) $V=\pi(e-2)$ **57.** $V=2\pi^2$ **59.** $\pi^3-6\pi$

61. (a) $-\frac{1}{4}\sin^3 x\cos x-\frac{3}{8}\sin x\cos x+\frac{3}{8}x+C$ (b) $8/15$

65. (a) $\frac{1}{3}\tan^3 x-\tan x+x+C$ (b) $\frac{1}{3}\sec^2 x\tan x+\frac{2}{3}\tan x+C$
(c) $x^3e^x-3x^2e^x+6xe^x-6e^x+C$

69. $(x+1)\ln(x+1)-x+C$ **71.** $\frac{1}{2}(x^2+1)\tan^{-1}x-\frac{1}{2}x+C$

Exercise Set 7.3 (Page 506)

1. $-\frac{1}{4}\cos^4 x+C$ **3.** $\frac{\theta}{2}-\frac{1}{20}\sin 10\theta+C$

5. $\frac{1}{3a}\cos^3 a\theta-\cos a\theta+C$ **7.** $\frac{1}{2a}\sin^2 ax+C$

9. $\frac{1}{3}\sin^3 t-\frac{1}{5}\sin^5 t+C$ **11.** $\frac{1}{8}x-\frac{1}{32}\sin 4x+C$

13. $-\frac{1}{10}\cos 5x+\frac{1}{2}\cos x+C$ **15.** $-\frac{1}{3}\cos(3x/2)-\cos(x/2)+C$

17. $2/3$ **19.** 0 **21.** $7/24$ **23.** $\frac{1}{2}\tan(2x-1)+C$

25. $\ln|\cos(e^{-x})|+C$ **27.** $\frac{1}{4}\ln|\sec 4x+\tan 4x|+C$

29. $\frac{1}{3}\tan^3 x+C$ **31.** $\frac{1}{16}\sec^4 4x+C$ **33.** $\frac{1}{7}\sec^7 x-\frac{1}{5}\sec^5 x+C$

35. $\frac{1}{4}\sec^3 x\tan x-\frac{5}{8}\sec x\tan x+\frac{3}{8}\ln|\sec x+\tan x|+C$

37. $\frac{1}{3}\sec^3 t+C$ **39.** $\tan x+\frac{1}{3}\tan^3 x+C$

41. $\frac{1}{8}\tan^2 4x+\frac{1}{4}\ln|\cos 4x|+C$ **43.** $\frac{2}{3}\tan^{3/2}x+\frac{2}{7}\tan^{7/2}x+C$

45. $\frac{1}{2}-\frac{\pi}{8}$ **47.** $-\frac{1}{2}+\ln 2$ **49.** $-\frac{1}{5}\csc^5 x+\frac{1}{3}\csc^3 x+C$

51. $-\frac{1}{2}\csc^2 x-\ln|\sin x|+C$

Responses to True–False questions may be abridged to save space.

53. True; $\int\sin^5 x\cos^8 x\,dx=\int\sin x(1-\cos^2 x)^2\cos^8 x\,dx=$
$-\int(1-u^2)^2u^8\,du=-\int(u^8-2u^{10}+u^{12})\,du$

55. False; use this identity to help evaluate integrals of the form
$\int\sin mx\cos nx\,dx$.

59. $L=\ln(\sqrt{2}+1)$ **61.** $V=\pi/2$

67. $-\dfrac{1}{\sqrt{a^2+b^2}} \ln \left[\dfrac{\sqrt{a^2+b^2} + a\cos x - b\sin x}{a\sin x + b\cos x} \right] + C$

69. (a) $\frac{2}{3}$ (b) $3\pi/16$ (c) $\frac{8}{15}$ (d) $5\pi/32$

Integration by Part (Stewart)

7.1 Exercises

1–2 Evaluate the integral using integration by parts with the indicated choices of u and dv.

1. $\int x^2 \ln x\, dx; \quad u = \ln x,\ dv = x^2\, dx$

2. $\int \theta \cos\theta\, d\theta; \quad u = \theta,\ dv = \cos\theta\, d\theta$

3–36 Evaluate the integral.

3. $\int x \cos 5x\, dx$

4. $\int y e^{0.2y}\, dy$

5. $\int t e^{-3t}\, dt$

6. $\int (x-1) \sin \pi x\, dx$

7. $\int (x^2 + 2x) \cos x\, dx$

8. $\int t^2 \sin \beta t\, dt$

9. $\int \ln \sqrt[3]{x}\, dx$

10. $\int \sin^{-1} x\, dx$

11. $\int \arctan 4t\, dt$

12. $\int p^5 \ln p\, dp$

13. $\int t \sec^2 2t\, dt$

14. $\int s 2^s\, ds$

15. $\int (\ln x)^2\, dx$

16. $\int t \sinh mt\, dt$

17. $\int e^{2\theta} \sin 3\theta\, d\theta$

18. $\int e^{-\theta} \cos 2\theta\, d\theta$

19. $\int z^3 e^z\, dz$

20. $\int x \tan^2 x\, dx$

21. $\int \dfrac{xe^{2x}}{(1+2x)^2}\, dx$

22. $\int (\arcsin x)^2\, dx$

23. $\int_0^{1/2} x \cos \pi x\, dx$

24. $\int_0^1 (x^2+1) e^{-x}\, dx$

25. $\int_0^1 t \cosh t\, dt$

26. $\int_4^9 \dfrac{\ln y}{\sqrt{y}}\, dy$

27. $\int_1^3 r^3 \ln r\, dr$

28. $\int_0^{2\pi} t^2 \sin 2t\, dt$

Graphing calculator or computer required   1. Homework Hints available at stewartcalculus.com

29. $\displaystyle\int_0^1 \frac{y}{e^{2y}}\,dy$

30. $\displaystyle\int_1^{\sqrt{3}} \arctan(1/x)\,dx$

31. $\displaystyle\int_0^{1/2} \cos^{-1}x\,dx$

32. $\displaystyle\int_1^2 \frac{(\ln x)^2}{x^3}\,dx$

33. $\displaystyle\int \cos x\,\ln(\sin x)\,dx$

34. $\displaystyle\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr$

35. $\displaystyle\int_1^2 x^4(\ln x)^2\,dx$

36. $\displaystyle\int_0^t e^s \sin(t-s)\,ds$

37–42 First make a substitution and then use integration by parts to evaluate the integral.

37. $\displaystyle\int \cos\sqrt{x}\,dx$

38. $\displaystyle\int t^3 e^{-t^2}\,dt$

39. $\displaystyle\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3\cos(\theta^2)\,d\theta$

40. $\displaystyle\int_0^{\pi} e^{\cos t}\sin 2t\,dt$

41. $\displaystyle\int x\ln(1+x)\,dx$

42. $\displaystyle\int \sin(\ln x)\,dx$

43–46 Evaluate the indefinite integral. Illustrate, and check that your answer is reasonable, by graphing both the function and its antiderivative (take $C=0$).

43. $\displaystyle\int xe^{-2x}\,dx$

44. $\displaystyle\int x^{3/2}\ln x\,dx$

45. $\displaystyle\int x^3\sqrt{1+x^2}\,dx$

46. $\displaystyle\int x^2\sin 2x\,dx$

47. (a) Use the reduction formula in Example 6 to show that

$$\int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

(b) Use part (a) and the reduction formula to evaluate $\int \sin^4 x\,dx$.

48. (a) Prove the reduction formula

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$$

(b) Use part (a) to evaluate $\int \cos^2 x\,dx$.
(c) Use parts (a) and (b) to evaluate $\int \cos^4 x\,dx$.

49. (a) Use the reduction formula in Example 6 to show that

$$\int_0^{\pi/2} \sin^n x\,dx = \frac{n-1}{n}\int_0^{\pi/2} \sin^{n-2}x\,dx$$

where $n \geqslant 2$ is an integer.
(b) Use part (a) to evaluate $\int_0^{\pi/2} \sin^3 x\,dx$ and $\int_0^{\pi/2} \sin^5 x\,dx$.
(c) Use part (a) to show that, for odd powers of sine,

$$\int_0^{\pi/2} \sin^{2n+1}x\,dx = \frac{2\cdot 4\cdot 6\cdot\cdots\cdot 2n}{3\cdot 5\cdot 7\cdot\cdots\cdot(2n+1)}$$

50. Prove that, for even powers of sine,

$$\int_0^{\pi/2} \sin^{2n}x\,dx = \frac{1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)}{2\cdot 4\cdot 6\cdot\cdots\cdot 2n}\,\frac{\pi}{2}$$

51–54 Use integration by parts to prove the reduction formula.

51. $\displaystyle\int (\ln x)^n\,dx = x(\ln x)^n - n\int (\ln x)^{n-1}\,dx$

52. $\displaystyle\int x^n e^x\,dx = x^n e^x - n\int x^{n-1}e^x\,dx$

53. $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx \quad (n \neq 1)$

54. $\displaystyle\int \sec^n x\,dx = \frac{\tan x\sec^{n-2}x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx \quad (n \neq 1)$

55. Use Exercise 51 to find $\int (\ln x)^3\,dx$.

56. Use Exercise 52 to find $\int x^4 e^x\,dx$.

57–58 Find the area of the region bounded by the given curves.

57. $y = x^2\ln x,\quad y = 4\ln x$

58. $y = x^2 e^{-x},\quad y = xe^{-x}$

59–60 Use a graph to find approximate x-coordinates of the points of intersection of the given curves. Then find (approximately) the area of the region bounded by the curves.

59. $y = \arcsin(\frac{1}{2}x),\quad y = 2 - x^2$

60. $y = x\ln(x+1),\quad y = 3x - x^2$

61–63 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis.

61. $y = \cos(\pi x/2),\ y = 0,\ 0 \leqslant x \leqslant 1;\quad$ about the y-axis

62. $y = e^x,\ y = e^{-x},\ x = 1;\quad$ about the y-axis

63. $y = e^{-x},\ y = 0,\ x = -1,\ x = 0;\quad$ about $x = 1$

64. Calculate the volume generated by rotating the region bounded by the curves $y = \ln x$, $y = 0$, and $x = 2$ about each axis.
(a) the y-axis
(b) the x-axis

65. Calculate the average value of $f(x) = x\sec^2 x$ on the interval $[0, \pi/4]$.

66. A rocket accelerates by burning its onboard fuel, so its mass decreases with time. Suppose the initial mass of the rocket at liftoff (including its fuel) is m, the fuel is consumed at rate r, and the exhaust gases are ejected with constant velocity v_e (relative to the rocket). A model for the velocity of the rocket at time t is given by the equation

$$v(t) = -gt - v_e \ln \frac{m - rt}{m}$$

where g is the acceleration due to gravity and t is not too large. If $g = 9.8 \text{ m/s}^2$, $m = 30{,}000$ kg, $r = 160$ kg/s, and $v_e = 3000$ m/s, find the height of the rocket one minute after liftoff.

67. A particle that moves along a straight line has velocity $v(t) = t^2e^{-t}$ meters per second after t seconds. How far will it travel during the first t seconds?

68. If $f(0) = g(0) = 0$ and f'' and g'' are continuous, show that

$$\int_0^a f(x)g''(x)\,dx = f(a)g'(a) - f'(a)g(a) + \int_0^a f''(x)g(x)\,dx$$

69. Suppose that $f(1) = 2$, $f(4) = 7$, $f'(1) = 5$, $f'(4) = 3$, and f'' is continuous. Find the value of $\int_1^4 xf''(x)\,dx$.

70. (a) Use integration by parts to show that

$$\int f(x)\,dx = xf(x) - \int xf'(x)\,dx$$

(b) If f and g are inverse functions and f' is continuous, prove that

$$\int_a^b f(x)\,dx = bf(b) - af(a) - \int_{f(a)}^{f(b)} g(y)\,dy$$

[*Hint:* Use part (a) and make the substitution $y = f(x)$.]

(c) In the case where f and g are positive functions and $b > a > 0$, draw a diagram to give a geometric interpretation of part (b).

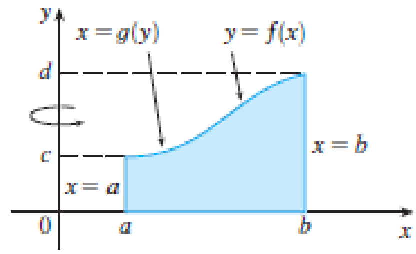
(d) Use part (b) to evaluate $\int_1^e \ln x\,dx$.

71. We arrived at Formula 6.3.2, $V = \int_a^b 2\pi x f(x)\,dx$, by using cylindrical shells, but now we can use integration by parts to prove it using the slicing method of Section 6.2, at least for the case where f is one-to-one and therefore has an inverse function g. Use the figure to show that

$$V = \pi b^2 d - \pi a^2 c - \int_c^d \pi[g(y)]^2\,dy$$

Make the substitution $y = f(x)$ and then use integration by parts on the resulting integral to prove that

$$V = \int_a^b 2\pi x f(x)\,dx$$



72. Let $I_n = \int_0^{\pi/2} \sin^n x\,dx$.

(a) Show that $I_{2n+2} \leqslant I_{2n+1} \leqslant I_{2n}$.

(b) Use Exercise 50 to show that

$$\frac{I_{2n+2}}{I_{2n}} = \frac{2n+1}{2n+2}$$

(c) Use parts (a) and (b) to show that

$$\frac{2n+1}{2n+2} \leqslant \frac{I_{2n+1}}{I_{2n}} \leqslant 1$$

and deduce that $\lim_{n\to\infty} I_{2n+1}/I_{2n} = 1$.

(d) Use part (c) and Exercises 49 and 50 to show that

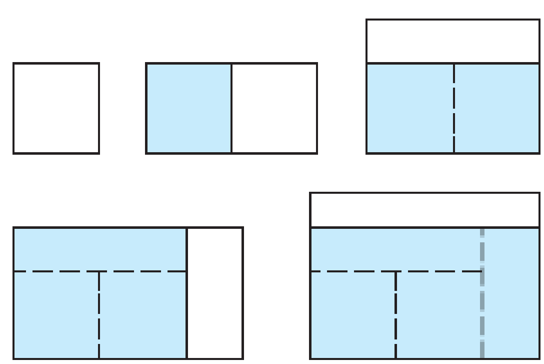
$$\lim_{n\to\infty} \frac{2}{1}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\frac{4}{5}\cdot\frac{6}{5}\cdot\frac{6}{7}\cdot\cdots\cdot\frac{2n}{2n-1}\cdot\frac{2n}{2n+1} = \frac{\pi}{2}$$

This formula is usually written as an infinite product:

$$\frac{\pi}{2} = \frac{2}{1}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\frac{4}{5}\cdot\frac{6}{5}\cdot\frac{6}{7}\cdot\cdots$$

and is called the *Wallis product*.

(e) We construct rectangles as follows. Start with a square of area 1 and attach rectangles of area 1 alternately beside or on top of the previous rectangle (see the figure). Find the limit of the ratios of width to height of these rectangles.



7.2 Exercises

1–49 Evaluate the integral.

1. $\int \sin^2 x \cos^3 x \, dx$

2. $\int \sin^3\theta \cos^4\theta \, d\theta$

3. $\int_0^{\pi/2} \sin^7\theta \cos^5\theta \, d\theta$

4. $\int_0^{\pi/2} \sin^5 x \, dx$

5. $\int \sin^2(\pi x) \cos^5(\pi x) \, dx$

6. $\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}} \, dx$

7. $\int_0^{\pi/2} \cos^2\theta \, d\theta$

8. $\int_0^{2\pi} \sin^2\left(\frac{1}{3}\theta\right) d\theta$

9. $\int_0^{\pi} \cos^4(2t) \, dt$

10. $\int_0^{\pi} \sin^2 t \cos^4 t \, dt$

11. $\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$

12. $\int_0^{\pi/2} (2 - \sin\theta)^2 \, d\theta$

13. $\int t \sin^2 t \, dt$

14. $\int \cos\theta \cos^5(\sin\theta) \, d\theta$

15. $\int \frac{\cos^5\alpha}{\sqrt{\sin\alpha}} \, d\alpha$

16. $\int x \sin^3 x \, dx$

17. $\int \cos^2 x \tan^3 x\, dx$

18. $\int \cot^5\theta \sin^4\theta\, d\theta$

19. $\int \frac{\cos x + \sin 2x}{\sin x}\, dx$

20. $\int \cos^2 x \sin 2x\, dx$

21. $\int \tan x \sec^3 x\, dx$

22. $\int \tan^2\theta \sec^4\theta\, d\theta$

23. $\int \tan^2 x\, dx$

24. $\int (\tan^2 x + \tan^4 x)\, dx$

25. $\int \tan^4 x \sec^6 x\, dx$

26. $\int_0^{\pi/4} \sec^4\theta \tan^4\theta\, d\theta$

27. $\int_0^{\pi/3} \tan^5 x \sec^4 x\, dx$

28. $\int \tan^5 x \sec^3 x\, dx$

29. $\int \tan^3 x \sec x\, dx$

30. $\int_0^{\pi/4} \tan^4 t\, dt$

31. $\int \tan^5 x\, dx$

32. $\int \tan^2 x \sec x\, dx$

33. $\int x \sec x \tan x\, dx$

34. $\int \frac{\sin\phi}{\cos^3\phi}\, d\phi$

35. $\int_{\pi/6}^{\pi/2} \cot^2 x\, dx$

36. $\int_{\pi/4}^{\pi/2} \cot^3 x\, dx$

37. $\int_{\pi/4}^{\pi/2} \cot^5\phi \csc^3\phi\, d\phi$

38. $\int \csc^4 x \cot^6 x\, dx$

39. $\int \csc x\, dx$

40. $\int_{\pi/6}^{\pi/3} \csc^3 x\, dx$

41. $\int \sin 8x \cos 5x\, dx$

42. $\int \cos \pi x \cos 4\pi x\, dx$

43. $\int \sin 5\theta \sin\theta\, d\theta$

44. $\int \frac{\cos x + \sin x}{\sin 2x}\, dx$

45. $\int_0^{\pi/6} \sqrt{1 + \cos 2x}\, dx$

46. $\int_0^{\pi/4} \sqrt{1 - \cos 4\theta}\, d\theta$

47. $\int \frac{1 - \tan^2 x}{\sec^2 x}\, dx$

48. $\int \frac{dx}{\cos x - 1}$

49. $\int x \tan^2 x\, dx$

50. If $\int_0^{\pi/4} \tan^6 x \sec x\, dx = I$, express the value of $\int_0^{\pi/4} \tan^8 x \sec x\, dx$ in terms of I.

51–54 Evaluate the indefinite integral. Illustrate, and check that your answer is reasonable, by graphing both the integrand and its antiderivative (taking $C = 0$).

51. $\int x \sin^2(x^2)\, dx$

52. $\int \sin^5 x \cos^3 x\, dx$

53. $\int \sin 3x \sin 6x\, dx$

54. $\int \sec^4 \frac{x}{2}\, dx$

55. Find the average value of the function $f(x) = \sin^2 x \cos^3 x$ on the interval $[-\pi, \pi]$.

56. Evaluate $\int \sin x \cos x\, dx$ by four methods:
   (a) the substitution $u = \cos x$
   (b) the substitution $u = \sin x$
   (c) the identity $\sin 2x = 2 \sin x \cos x$
   (d) integration by parts

   Explain the different appearances of the answers.

57–58 Find the area of the region bounded by the given curves.

57. $y = \sin^2 x$, $y = \cos^2 x$, $-\pi/4 \leqslant x \leqslant \pi/4$

58. $y = \sin^3 x$, $y = \cos^3 x$, $\pi/4 \leqslant x \leqslant 5\pi/4$

59–60 Use a graph of the integrand to guess the value of the integral. Then use the methods of this section to prove that your guess is correct.

59. $\int_0^{2\pi} \cos^3 x\, dx$

60. $\int_0^2 \sin 2\pi x \cos 5\pi x\, dx$

61–64 Find the volume obtained by rotating the region bounded by the given curves about the specified axis.

61. $y = \sin x$, $y = 0$, $\pi/2 \leqslant x \leqslant \pi$; about the x-axis

62. $y = \sin^2 x$, $y = 0$, $0 \leqslant x \leqslant \pi$; about the x-axis

63. $y = \sin x$, $y = \cos x$, $0 \leqslant x \leqslant \pi/4$; about $y = 1$

64. $y = \sec x$, $y = \cos x$, $0 \leqslant x \leqslant \pi/3$; about $y = -1$

65. A particle moves on a straight line with velocity function $v(t) = \sin \omega t \cos^2 \omega t$. Find its position function $s = f(t)$ if $f(0) = 0$.

66. Household electricity is supplied in the form of alternating current that varies from 155 V to -155 V with a frequency of 60 cycles per second (Hz). The voltage is thus given by the equation

$$E(t) = 155 \sin(120\pi t)$$

where t is the time in seconds. Voltmeters read the RMS (root-mean-square) voltage, which is the square root of the average value of $[E(t)]^2$ over one cycle.
   (a) Calculate the RMS voltage of household current.
   (b) Many electric stoves require an RMS voltage of 220 V. Find the corresponding amplitude A needed for the voltage $E(t) = A \sin(120\pi t)$.

67–69 Prove the formula, where m and n are positive integers.

67. $\int_{-\pi}^{\pi} \sin mx \cos nx \, dx = 0$

68. $\int_{-\pi}^{\pi} \sin mx \sin nx \, dx = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$

69. $\int_{-\pi}^{\pi} \cos mx \cos nx \, dx = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$

70. A *finite Fourier series* is given by the sum

$$\begin{aligned} f(x) &= \sum_{n=1}^{N} a_n \sin nx \\ &= a_1 \sin x + a_2 \sin 2x + \cdots + a_N \sin Nx \end{aligned}$$

Show that the mth coefficient a_m is given by the formula

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin mx \, dx$$

Answers

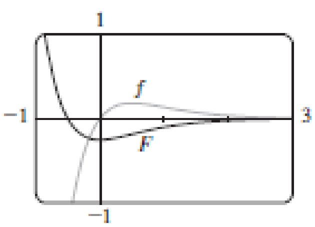
PROBLEMS PLUS ▪ PAGE 459

1. (a) $f(t) = 3t^2$ (b) $f(x) = \sqrt{2x/\pi}$ **3.** $\frac{32}{27}$
5. (b) 0.2261 (c) 0.6736 m
(d) (i) $1/(105\pi) \approx 0.003$ in/s (ii) $370\pi/3$ s $\approx$ 6.5 min
9. $y = \frac{32}{9}x^2$
11. (a) $V = \int_0^h \pi[f(y)]^2\,dy$
(c) $f(y) = \sqrt{kA/(\pi C)}\,y^{1/4}$. Advantage: the markings on the container are equally spaced.
13. $b = 2a$ **15.** $B = 16A$

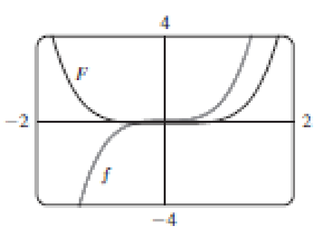
CHAPTER 7

EXERCISES 7.1 ▪ PAGE 468

1. $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$ **3.** $\frac{1}{5}x \sin 5x + \frac{1}{25}\cos 5x + C$
5. $-\frac{1}{3}te^{-3t} - \frac{1}{9}e^{-3t} + C$
7. $(x^2 + 2x)\sin x + (2x + 2)\cos x - 2\sin x + C$
9. $x \ln \sqrt[3]{x} - \frac{1}{3}x + C$ **11.** $t \arctan 4t - \frac{1}{8}\ln(1 + 16t^2) + C$
13. $\frac{1}{2}t \tan 2t - \frac{1}{4}\ln|\sec 2t| + C$
15. $x(\ln x)^2 - 2x \ln x + 2x + C$
17. $\frac{1}{13}e^{2\theta}(2\sin 3\theta - 3\cos 3\theta) + C$
19. $z^3e^z - 3z^2e^z + 6ze^z - 6e^z + C$
21. $\dfrac{e^{2x}}{4(2x+1)} + C$ **23.** $\dfrac{\pi - 2}{2\pi^2}$
25. $1 - 1/e$ **27.** $\frac{81}{4}\ln 3 - 5$ **29.** $\frac{1}{4} - \frac{3}{4}e^{-2}$
31. $\frac{1}{6}\left(\pi + 6 - 3\sqrt{3}\right)$ **33.** $\sin x\,(\ln \sin x - 1) + C$
35. $\frac{32}{5}(\ln 2)^2 - \frac{64}{25}\ln 2 + \frac{62}{125}$
37. $2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$ **39.** $-\frac{1}{2} - \pi/4$
41. $\frac{1}{2}(x^2 - 1)\ln(1 + x) - \frac{1}{4}x^2 + \frac{1}{2}x + \frac{3}{4} + C$
43. $-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$



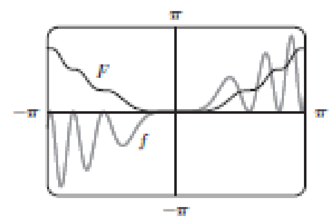
45. $\frac{1}{3}x^2(1 + x^2)^{3/2} - \frac{2}{15}(1 + x^2)^{5/2} + C$



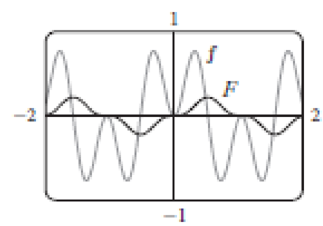
47. (b) $-\frac{1}{4}\cos x \sin^3 x + \frac{3}{8}x - \frac{3}{16}\sin 2x + C$
49. (b) $\frac{2}{3}, \frac{8}{15}$
55. $x[(\ln x)^3 - 3(\ln x)^2 + 6\ln x - 6] + C$
57. $\frac{16}{3}\ln 2 - \frac{29}{9}$ **59.** $-1.75119, 1.17210; 3.99926$
61. $4 - 8/\pi$ **63.** $2\pi e$ **65.** $1 - (2/\pi)\ln 2$
67. $2 - e^{-t}(t^2 + 2t + 2)$ m **69.** 2

EXERCISES 7.2 ▪ PAGE 476

1. $\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$ **3.** $\frac{1}{120}$
5. $\dfrac{1}{3\pi}\sin^3(\pi x) - \dfrac{2}{5\pi}\sin^5(\pi x) + \dfrac{1}{7\pi}\sin^7(\pi x) + C$
7. $\pi/4$ **9.** $3\pi/8$ **11.** $\pi/16$
13. $\frac{1}{4}t^2 - \frac{1}{4}t \sin 2t - \frac{1}{8}\cos 2t + C$
15. $\frac{2}{45}\sqrt{\sin\alpha}\,(45 - 18\sin^2\alpha + 5\sin^4\alpha) + C$
17. $\frac{1}{2}\cos^2 x - \ln|\cos x| + C$ **19.** $\ln|\sin x| + 2\sin x + C$
21. $\frac{1}{3}\sec^3 x + C$ **23.** $\tan x - x + C$
25. $\frac{1}{9}\tan^9 x + \frac{2}{7}\tan^7 x + \frac{1}{5}\tan^5 x + C$ **27.** $\frac{117}{8}$
29. $\frac{1}{3}\sec^3 x - \sec x + C$
31. $\frac{1}{4}\sec^4 x - \tan^2 x + \ln|\sec x| + C$
33. $x \sec x - \ln|\sec x + \tan x| + C$ **35.** $\sqrt{3} - \frac{1}{3}\pi$
37. $\frac{22}{105}\sqrt{2} - \frac{8}{105}$ **39.** $\ln|\csc x - \cot x| + C$
41. $-\frac{1}{6}\cos 3x - \frac{1}{26}\cos 13x + C$ **43.** $\frac{1}{8}\sin 4\theta - \frac{1}{12}\sin 6\theta + C$
45. $\frac{1}{2}\sqrt{2}$ **47.** $\frac{1}{2}\sin 2x + C$
49. $x \tan x - \ln|\sec x| - \frac{1}{2}x^2 + C$
51. $\frac{1}{4}x^2 - \frac{1}{4}\sin(x^2)\cos(x^2) + C$



53. $\frac{1}{6}\sin 3x - \frac{1}{18}\sin 9x + C$



55. 0 **57.** 1 **59.** 0 **61.** $\pi^2/4$ **63.** $\pi\left(2\sqrt{2} - \frac{5}{2}\right)$
65. $s = (1 - \cos^3\omega t)/(3\omega)$

EXERCISES 7.3 ▪ PAGE 483

1. $-\dfrac{\sqrt{4 - x^2}}{4x} + C$ **3.** $\sqrt{x^2 - 4} - 2\sec^{-1}\left(\dfrac{x}{2}\right) + C$
5. $\dfrac{\pi}{24} + \dfrac{\sqrt{3}}{8} - \dfrac{1}{4}$ **7.** $\dfrac{1}{\sqrt{2}a^2}$
9. $\ln\left(\sqrt{x^2 + 16} + x\right) + C$ **11.** $\frac{1}{4}\sin^{-1}(2x) + \frac{1}{2}x\sqrt{1 - 4x^2} + C$
13. $\frac{1}{6}\sec^{-1}(x/3) - \sqrt{x^2 - 9}/(2x^2) + C$
15. $\frac{1}{16}\pi a^4$ **17.** $\sqrt{x^2 - 7} + C$
19. $\ln\left|\left(\sqrt{1 + x^2} - 1\right)/x\right| + \sqrt{1 + x^2} + C$ **21.** $\frac{9}{500}\pi$
23. $\frac{9}{2}\sin^{-1}((x - 2)/3) + \frac{1}{2}(x - 2)\sqrt{5 + 4x - x^2} + C$
25. $\sqrt{x^2 + x + 1} - \frac{1}{2}\ln\left(\sqrt{x^2 + x + 1} + x + \frac{1}{2}\right) + C$
27. $\frac{1}{2}(x + 1)\sqrt{x^2 + 2x} - \frac{1}{2}\ln\left|x + 1 + \sqrt{x^2 + 2x}\right| + C$
29. $\frac{1}{4}\sin^{-1}(x^2) + \frac{1}{4}x^2\sqrt{1 - x^4} + C$
33. $\frac{1}{6}\left(\sqrt{48} - \sec^{-1} 7\right)$ **37.** $\frac{3}{8}\pi^2 + \frac{3}{4}\pi$
41. $2\pi^2Rr^2$ **43.** $r\sqrt{R^2 - r^2} + \pi r^2/2 - R^2 \arcsin(r/R)$